II. *A Method for determining the Number of impossible Roots in adfected Æquations.* By Mr. George Campbell.

LEMMA I.

IN every adfected quadratick Æquation $a x^2 - B x + A = 0$, whose Roots are real, a fourth Part of the Square of the Coefficient of the second Term is greater than the Rectangle under the Coefficient of the first Term and the absolute Number or $\frac{1}{4} B^2 \searrow a \times A$; and *vice versa* if $\frac{1}{4} B^2 \searrow a \times A$, the Roots of the Æquation $a x^2 - B x + A = 0$, will be real. But if $\frac{1}{4} B^2 \nwarrow a \times A$, the Roots will be impossible. This is evident from the Roots of the Æquation being $\dfrac{\frac{1}{2} B + \sqrt{\frac{1}{4} B^2 - a \times A}}{a}$,

$\dfrac{\frac{1}{2} B - \sqrt{\frac{1}{4} B^2 - a \times A}}{a}$.

LEMMA II.

Whatever be the Number of impossible Roots in the Æquation $x^n - B x^{n-1} + C x^{n-2} - D x^{n-3} + \&c.$ $\pm d x^3 \mp c x^2 \pm b x \mp A = 0$, there are just as many in the Æquation $A x^n - b x^{n-1} + c x^{n-2} - d x^{n-3} + \&c. \pm D x^3 \mp C x^2 \pm B x \mp 1 = 0$. For the Roots of the last Æquation are the Reciprocals of those of the first, as is evident from common Algebra. Let the Roots of the biquadratick Æquation $x^4 - B x^3 + C x^2 - D x + A = 0$ be a, b, c, d, whereof let c, d be impossible, then the Roots of the Æquation

Z z z z $\qquad$ $A x^4 -$

$A x^4 - D x^3 + C x^2 - B x + 1 = 0$ will be

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d},$ and therefore two of them to wit $\frac{1}{c}, \frac{1}{d}$ impoſſible.

LEMMA III.

In every Æquation $x^n - B x^{n-1} + C x^{n-2} - D x^{n-3} + E x^{n-4} - \&c. \pm e x^4 \mp d x^3 \pm c x^2 \mp b x \pm A = 0$, all whoſe Roots are real, if each Term be multiply'd by the Index of x in that Term, and each Product be divided by x, the reſulting Æquation $n x^{n-1} - \overline{n-1} B x^{n-2} + \overline{n-2} C x^{n-3} - \overline{n-3} D x^{n-4} + \overline{n-4} E x^{n-5} - \&c. \pm 4 e x^3 \mp 3 d x^2 \pm 2 c x \mp b = 0$ ſhall have all its Roots real. Thus if all the Roots of the Æquation $x^4 - B x^3 + C x^2 - D x + A = 0$ be real, then all the Roots of the Æquation $4 x^3 - 3 B x^2 + 2 C x - D = 0$ will alſo be real. This Lemma doth not hold converſly, for there are an Infinity of Caſes where all the Roots of the Æquation $n x^{n-1} - \overline{n-1} B x^{n-2} + \overline{n-2} C x^{n-3} - \overline{n-3} D x^{n-4} + \&c. \pm 3 d x^2 \mp 2 c x \pm b = 0$ are real, at the ſame Time ſome or perhaps all the Roots of the Æquation $x^n - B x^{n-1} + C x^{n-2} - D x^{n-3} + \&c. \pm d x^3 \mp c x^2 \pm b x \mp A = 0$ are impoſſible: But whatever be the Number of impoſſible Roots in the Æquation $n x^{n-1} - \overline{n-1} B x^{n-2} + \overline{n-2} C x^{n-3} - \&c. \pm 2 c x \mp b = 0,$ there are at leaſt as many in the Æquation $x^n - B x^{n-1} + C x^{n-2} \&c. \pm c x^2 \mp b x \pm A = 0.$ Thus all the Roots of the Æquation $4 x^3 - 3 B x^2 + 2 C x - D = 0$ may be real, and yet two or perhaps all the four

four Roots of the Æquation $x^4 - B x^3 + C x^2 - D x + A = 0$ may be impoffible, but if two of the Roots of the Æquation $4 x^3 - 3 B x^2 + 2 C x - D = 0$ be impoffible, there muft be at leaft two impoffible Roots in the Æquation $x^4 - B x^3 + C x^2 - D x + A = 0$. All this hath been demonftrated by Algebraical Writers, particularly by Mr. *Reyneau* in his *Analyfe Demontré*, and is eafily made evident by the Method of the *Maxima* and *Minima*.

COROLARY. Let all the Roots of the Æquation $x^n - B x^{n-1} + C x^{n-2} - D x^{n-3} + E x^{n-4} - F x^{n-5} + \&c. \pm f x^5 \mp e x^4 \pm d x^3 \mp c x^2 \pm b x \mp A = 0$ be real, and by this Lemma all the Roots of the Æquation $n x^{n-1} - \overline{n-1} B x^{n-2} + \overline{n-2} C x^{n-3} - \overline{n-3} D x^{n-4} + \overline{n-4} E x^{n-5} - \overline{n-5} F x^{n-6} + \&c. \pm 5 f x^4 \mp 4 e x^3 \pm 3 d x^2 \mp 2 c x \pm b = 0$ will be real, and therefore (by the fame Lemma) all the Roots of the Æquation $n \times \overline{n-1} x^{n-2} - \overline{n-1} \times \overline{n-2} B x^{n-3} + \overline{n-2} \times \overline{n-3} C x^{n-4} - \overline{n-3} \times \overline{n-4} D x^{n-5} + \overline{n-4} \times \overline{n-5} E x^{n-6} - \overline{n-5} \times \overline{n-6} F x^{n-7} + \&c. \pm 20 f x^3 \mp 12 e x^2 \pm 6 d x \mp 2 c = 0$

or (dividing all by 2) of $n \times \dfrac{\overline{n-1}}{2} x^{n-2} - \overline{n-1} \times \dfrac{\overline{n-2}}{2} B x^{n-3} + \overline{n-2} \times \dfrac{\overline{n-3}}{2} C x^{n-4} - \&c. \pm 10 f x^3 \mp 6 e x^2 \pm 3 d x \mp c = 0$ will be real. After the fame Manner all the Roots of the Æquation

$n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} x^{n-3} - \overline{n-1} \times \dfrac{\overline{n-2}}{2} \times \dfrac{\overline{n-3}}{3} B x^{n-4} +$

$$B x^{n-4} + \overline{n-2} \times \frac{\overline{n-3}}{2} \times \frac{\overline{n-4}}{3} C x^{n-5} - \&c. \pm$$

$10 f x^2 \mp 4 e x \pm d = 0$ will be real; and thus we may defcend until we arrive at the quadratick Æqua-

tion $n \times \dfrac{\overline{n-1}}{2} x^2 - \overline{n-1} B x + C = 0$. The fame

Æquations do afcend thus $n \times \dfrac{\overline{n-1}}{2} x^2 - \overline{n-1} B x +$

$C = 0, \; n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} x^3 - \overline{n-1} \times \dfrac{\overline{n-2}}{2} B x^2 +$

$\overline{n-2} C x - D = 0, \; n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} \times \dfrac{\overline{n-3}}{4} x^4$

$\overline{n-1} \times \dfrac{\overline{n-2}}{2} \times \dfrac{\overline{n-3}}{3} B x^3 + \overline{n-2} \times \dfrac{\overline{n-3}}{2} \times$

$C x^2 - \overline{n-3} D x + E = 0, \; n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} \times$

$\dfrac{\overline{n-3}}{4} \times \dfrac{\overline{n-4}}{5} x^5 - \overline{n-1} \times \dfrac{\overline{n-2}}{2} \times \dfrac{\overline{n-3}}{3} \times \dfrac{\overline{n-4}}{4}$

$B x^4 + \overline{n-2} \times \dfrac{\overline{n-3}}{2} \times \dfrac{\overline{n-4}}{3} C x^3 - \overline{n-3} \times \dfrac{\overline{n-4}}{2}$

$D x^2 + \overline{n-4} E x - F = 0$, and fo on. Let M re-
prefent any of the Coefficients of the Æquation
$x^n - B x^{n-1} + C x^{n-2} - D x^{n-3} + E x^{n-4} - \&c.$
$\pm A = 0$, and let $L N$ be the adjacent Coefficients,
let M be the Exponent of the Coefficient M: By the
Exponent of a Coefficient I mean the Number which
expreffeth

expreffeth the Place which it hath among the Coeffi-
cients, thus if M reprefent the Coefficient E (and
therefore $L = \mathcal{D}$ and $N = F$) then $m = 4$. It
will be eafy to fee, that, amongft the foregoing
afcending Æquations, that which hath its abfolute

Number N will be $n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} \times \mathcal{C}c.$

$\dfrac{\overline{n-m}}{m+1} x^{m+1} - \overline{n-1} \times \dfrac{\overline{n-2}}{2} \times \mathcal{C}c. \dfrac{\overline{n-m}}{m} B x^{\overline{m}} +$

$\overline{n-2} \times \mathcal{C}c. \dfrac{\overline{n-m}}{m-1} C x^{m-1} - \mathcal{C}c. \pm \overline{n-m+1} \times$

$\dfrac{\overline{n-m}}{2} L x^2 \mp \overline{n-m} M x \pm N = 0$, all whofe Roots

are real when all the Roots of the Æquation
$x^n - B x^{n-1} + C x^{n-2} - \mathcal{C}c. \pm A = 0$ are real.
Let $N = F$ and therefore $M = E$, $L = \mathcal{D}$ and
$m = 4$, then that of the afcending Æquations whofe

abfolute Number is F, will be $n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} \times$

$\dfrac{\overline{n-3}}{4} \times \dfrac{\overline{n-4}}{5} x^5 - \overline{n-1} \times \dfrac{\overline{n-2}}{2} \times \dfrac{\overline{n-3}}{3} \times \dfrac{\overline{n-4}}{4}$

$B x^4 + \overline{n-2} \times \dfrac{\overline{n-3}}{2} \times \dfrac{\overline{n-4}}{3} C x^3 - \overline{n-3} \times \dfrac{\overline{n-4}}{2}$

$\mathcal{D} x^2 + \overline{n-4} E x - F = 0.$

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PROPOSITION I.

Let $x^n - B x^{n-1} + C x^{n-2} - D x^{n-3} + E x^{n-4} - \&c. \pm e x^4 \mp d x^3 \pm c x^2 \mp b x \pm A = 0$ be an Æquation of any Dimensions all whose Roots are real, let M be any Coefficient of this Æquation, L, N the adjacent Coefficients, and m the Exponent of M. Then the Square of any Coefficient M multi-ply'd by the Fraction $\dfrac{m \times \overline{n-m}}{\overline{m+1} \times \overline{n-m+1}}$ will al-ways exceed the Rectangle under the adjacent Coeffi-cients $L \times N$. Thus in the Æquation $x^4 - B x^3 + C x^2 - D x + A = 0$, where $n = 4$, making $M = C$ and therefore $L = B$, $N = D$, and $m = 2$, then $\dfrac{2 \times \overline{4-2}}{\overline{2+1} \times \overline{4-2+1}} \times C^2$ or $\dfrac{4}{9} C^2$ will exceed $B \times D$ providing all the Roots of the Æquation be real.

Because (by Lem. 3.) the Roots of the quadratick Æquation $n \times \dfrac{\overline{n-1}}{2} x^2 - \overline{n-1} B x + C = 0$, are real, therefore (by Lem. 1.) $\frac{1}{4}\overline{n-1}|^2 \times B^2$ must be greater than $n \times \dfrac{\overline{n-1}}{2} \times C$ and (dividing both by $n \times \dfrac{\overline{n-1}}{2}$) $\dfrac{\overline{n-1}}{2n} \times B^2$ greater than $1 \times C$. Therefore in the Æquation $x^n - B x^{n-1} + C x^{n-2} - D x^{n-3} + \&c. \pm A = 0$ of the n Degree, all whose Roots are real, the Square of B the Coefficient of the second Term,

Term, multiply'd by the Fraction $\dfrac{n-1}{2n}$ is greater than

$1 \times C$ the Rectangle under the adjacent Coefficients. But (by Lem. 2.) all the Roots of the Æquation $A x^n - b x^{n-1} + c x^{n-2} - \&c. \pm C x^2 \mp B x \pm$

$1 = 0$ or (dividing by A) of $x^n - \dfrac{b}{A} x^{n-1} +$

$\dfrac{c}{A} x^{n-2} - \&c. \pm \dfrac{C}{A} x^2 \mp \dfrac{B}{A} x \pm \dfrac{1}{A} = 0$ are real,

therefore (from what hath been juſt now ſaid) $\dfrac{\overline{n-1}}{2n} \times \dfrac{b^2}{A^2}$ muſt be greater than $1 \times \dfrac{c}{A}$ and conſe-

quently $\dfrac{\overline{n-1}}{2n} \times b^2$ greater than $c \times A$. Therefore

in an Æquation $x^n - B x^{n-1} + C x^{n-2} - \&c. + c x^2 \mp b x \pm A = 0$, of the n Degree, all whoſe Roots are real, the Square of the Coefficient of x

multiply'd by the Fraction $\dfrac{n-1}{2n}$ is greater than the

Rectangle under the Coefficient of x^2 and the abſolute Number. But by Cor. Lem. 3. all the Roots of the

Æquation $n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} \times \&c. \times \dfrac{\overline{n-m}}{m+1} x^{m+1} -$

$\overline{n-1} \times \dfrac{\overline{n-2}}{2} \times \&c. \times \dfrac{\overline{n-m}}{m} B x^m + \overline{n-2} \times \&c.$

$\dfrac{\overline{n-m}}{m-1} C x^{m-1} \&c. \pm \overline{n-m+1} \times \dfrac{\overline{n-m}}{2} \times L x^2 \mp$

$\overline{n-m}$

$\overline{n-m}\,M\,x \pm N = 0$ are real, therefore (feeing this Æquation is of the $m+1$ Degree) the Square of

$\overline{n-m} \times M$ multiply'd by the Fraction $\dfrac{\overline{m+1-1}}{2 \times \overline{m+1}}$

will be greater than the Rectangle under $\overline{n-m+1} \times$

$\dfrac{\overline{n-m}}{2} \times L$ and N, that is $\dfrac{m}{2 \times \overline{m+1}} \times \overline{n-m}\Big|^{2} \times$

M^{2} will be greater than $\overline{n-m+1} \times \dfrac{\overline{n-m}}{2} \times L \times N$

and therefore (dividing both by $\overline{n-m+1} \times \dfrac{\overline{n-m}}{2}$)

$\dfrac{m \times \overline{n-m}}{\overline{m+1} \times \overline{n-m+1}} \times M^{2}$ greater than $L \times N$.

COROLARY. Make a Series of Fractions $\dfrac{n}{1}, \dfrac{n-1}{2}, \dfrac{n-2}{3}, \dfrac{n-3}{4}$, &c. unto $\dfrac{1}{n}$ whofe Denominators are Numbers going on in the Progreffion 1, 2, 3, 4, &c. unto the Number n which is the Dimenfions of the Æquation $x^{n} - B\,x^{n-1} + C\,x^{n-2} -$ &c. $\pm A = 0$, and whofe Numerators are the fame Progreffion inverted. Divide the fecond of thefe Fractions by the firft, the third by the fecond, the fourth by the third, and fo on, and place the Fractions which refult from this Divifion above the middle Terms of

the Æquation, thus $x^{n} - B\,x^{n-1}\overset{\frac{n-1}{2n}}{} + C\,x^{n-2}\overset{\frac{2 \times \overline{n-2}}{3 \times \overline{n-1}}}{} - D\,x^{n-3} +$

$$\overset{\overset{\textstyle\frac{3\times n - 3}{}}{\frac{4\times n - 2}{}}}{\mathcal{D}} x^{n-3} + \overset{\overset{\textstyle\frac{4\times n - 4}{}}{\frac{5\times n - 3}{}}}{E} x^{n-4} - \&c. \pm A = 0.$$ Then if all
the Roots of the Æquation are real, the Square of
any Coefficient multiply'd by the Fraction which
ftands above, will be greater than the Rectangle un-
der the adjacent Coefficients. This Corolary doth not
hold converfly, for there are an Infinity of Æqua-
tions in which the Square of each Coefficient multi-
ply'd by the Fraction above it, may be greater than
the Rectangle under the adjacent Coefficients, and
notwithftanding fome or perhaps all of the Roots may
be impoffible. Therefore when the Square of a Coeffi-
cient multiply'd by the Fraction above, is greater than
the Rectangle under the adjacent Coefficients, from this
Circumftance nothing can be determined as to the
Poffibility or Impoffibility of the Roots of the Æqua-
tion : But when the Square of a Coefficient multiply'd
by the Fraction above it, is lefs than the Rectangle un-
der the adjacent Coefficients, it is a certain Indication
of two impoffible Roots. From what hath been faid,
is immediately deduced the Demonftration of that
Rule which the moft illuftrious *Newton* gives for de-
termining the Number of impoffible Roots in any gi-
ven Æquation.

SCHOLIUM.

Let the Roots of the Æquation $x^n - B x^{n-1} + C x^{n-2} - \mathcal{D} x^{n-3} + E x^{n-4} - F x^{n-5} + \&c. \pm A = 0$ (with their Signs) be reprefented by the Let-
ters $a, b, c, d, e, f, g, \&c.$ then (as is commonly
known) B will be the Sum of all the Roots or $= a + b + c + d + e + f + \&c.$ C the Sum of the Products

of

of all the Pairs of Roots or $= ab + ac + ad + af + ag +$ &c. D the Sum of the Products of all the Ternaryes of Roots or $= abc + abd + abe + abf + abg +$ &c. $E = abcd + abce + abcf + abeg +$ &c. $F = abcde + abcdf + abcdg + bcdef +$ &c. and so on. Let (as in this Proposition) M represent any of these Coefficients, L, N the adjacent Coefficients, and m the Exponent of M; let Z represent the Sum of the Squares of all the possible Differences between the Terms of the Coefficient M, let α be the Sum of all those of the foresaid Squares whose Terms differ by one Letter, β the Sum of all those Squares whose Terms differ by two Letters, γ the Sum of those Squares whose Terms differ by three Letters, δ the Sum of those Squares whose Terms differ by four Letters and so on. Thus if $M = F = abcde + abcdf + abcdg +$ &c.

then $Z = \overline{abcde - abcdf}|^2 + \overline{abcde - abcdg}|^2 + \overline{abcde - abcfg}|^2 + \overline{bcdef - abfgh}|^2 +$ &c.

$\alpha = \overline{abcde - abcdf}|^2 + \overline{abcde - abcdg}|^2 + \overline{abcde - abcdh}|^2 + \overline{bcdef - bcdeg}|^2 +$ &c.

$\beta = \overline{abcde - abcfg}|^2 + \overline{abcde - abcfh}|^2 + \overline{bcdef - acdfh}|^2 +$ &c. $\gamma = \overline{abcde - abfgh}|^2 + \overline{abcdf - abegh}|^2 +$ &c. $\delta = \overline{abcde - afghk}|^2 + \overline{acdfg - abehk}|^2 +$ &c. This being laid down I say that the Square of any Coefficient M multiply'd

by the Fraction $\dfrac{m \times \overline{n - m}}{\overline{m + 1} \times \overline{n - m + 1}}$ exceeds the Rect-angle under the adjacent Coefficients $L \times N$ by $\overline{n + 1} \times Z$

$$\frac{\overline{n+1} \times Z}{\overline{m+1} \times \overline{n-m+1}} - \frac{1}{2}\alpha - \frac{1}{3}\beta - \frac{1}{4}\gamma - \frac{1}{5}$$

δ — &c. The Series — $\frac{1}{2}\alpha - \frac{1}{3}\beta - \frac{1}{4}\gamma -$

&c. muſt conſiſt of m Number of Terms.

Let the Æquation be $x^5 - Bx^4 + Cx^3 - Dx^2 + Ex - A = o$, whoſe Roots let be a, b, c, d, e, in which Caſe $n = 5$. Let $M = B = a + b + c + d + e$, then $L = 1$, $N = G$, $m = 1$, $Z = \overline{a-b}|^2 + \overline{a-c}|^2 + \overline{a-d}|^2 + \overline{a-e}|^2 +$

$\overline{b-c}|^2 + $ &c. $= \alpha$; therefore $\dfrac{\overline{1 \times 5 - 1}}{\overline{1+1} \times \overline{5-1+1}} \times$

B^2 or $\dfrac{2}{5} B^2$ exceeds $1 \times C$ by $\dfrac{\overline{5+1} \times Z}{\overline{1+1} \times \overline{5-1+1}}$

$- \dfrac{1}{2}\alpha = \dfrac{3}{5}Z - \dfrac{1}{2}\alpha = $ (becauſe $Z = \alpha$)

$\dfrac{1}{10}Z = \dfrac{1}{10}\overline{a-b}|^2 + \dfrac{1}{10} \times \overline{a-c}|^2 + \dfrac{1}{10}\overline{a-d}|^2 +$

&c. which is always a poſitive Number when the Roots a, b, c, d, e are real, poſitive or negative Numbers. Let $M = C = ab + ac + ad + ae + bc +$ &c. then $L = B$, $N = D$, $m = 2$, $Z = \overline{ab-ac}|^2 + \overline{ab-ad}|^2 + \overline{ab-cd}|^2 + \overline{ab-de}|^2 + $ &c. $\alpha = \overline{ab-ac}|^2 + \overline{ab-ad}|^2 + \overline{ab-ae}|^2 + $ &c. $\beta = \overline{ab-cd}|^2 + \overline{ab-ce}|^2 +$

$\overline{ab-de}|^2 + $ &c. therefore $\dfrac{\overline{2 \times 5 - 2}}{\overline{2+1} \times \overline{5-2+1}} \times C^2$

or $\frac{1}{2} C^2$ ſurpaſſeth $B \times \mathcal{D}$ by $\dfrac{\overline{5 + 1} \times Z}{\overline{2 + 1} \times \overline{5 - 2 + 1}}$

$- \dfrac{1}{2} \alpha - \dfrac{1}{3} \beta = $ (becauſe $Z = \alpha + \beta$) $= \dfrac{1}{6}$

$\beta = \dfrac{1}{6} \times \overline{a\,b - c\,d}|^2 + \dfrac{1}{6} \overline{a\,b - c\,e}|^2 + \dfrac{1}{6} \times$

$\overline{a\,b - d\,e}|^2 + \&c.$ which is always a poſitive Number when the Roots a, b, c, d, e are real Numbers, poſitive or negative. Let $M = \mathcal{D} = a\,b\,c + a\,b\,d + a\,b\,e + a\,c\,d + a\,c\,e + \&c.$ then $L = C$, $N = E$, $m = 3$, $Z = \overline{a\,b\,c - a\,b\,d}|^2 + \overline{a\,b\,c - a\,b\,e}|^2 + \overline{a\,b\,c - a\,d\,e}|^2 + \&c.$ $\alpha = \overline{a\,b\,c - a\,b\,d}|^2 + \overline{a\,b\,c - a\,b\,e}|^2 + \overline{a\,b\,c - a\,c\,d}|^2 + \&c.$ $\beta = \overline{a\,b\,c - a\,d\,e}|^2 + \overline{a\,b\,c - c\,d\,e}|^2 + \overline{a\,b\,c - b\,d\,e}|^2 + \&c.$ $\gamma = 0$, therefore $\dfrac{\overline{3 \times 5 - 3}}{\overline{3 + 1} \times \overline{5 - 3 + 1}} \times$

$\mathcal{D}^2$ or $\dfrac{1}{2} \mathcal{D}^2$ exceeds $C \times E$ by $\dfrac{\overline{5 + 1}}{\overline{3 + 1} \times \overline{5 - 3 + 1}} \times$

$Z - \dfrac{1}{2} \alpha - \dfrac{1}{3} \beta = $ (becauſe $Z = \alpha + \beta$) $= \dfrac{1}{6} \times$

$\beta = \dfrac{1}{6} \times \overline{a\,b\,c - a\,d\,e}|^2 + \dfrac{1}{6} \times \overline{a\,b\,c - c\,d\,e}|^2 + \dfrac{1}{6}$

$\overline{a\,b\,c - b\,d\,e}|^2 + \&c.$ which is a poſitive Number when the Roots are real Numbers. Let $M = E = a\,b\,c\,d + a\,b\,c\,e + a\,b\,d\,e + b\,c\,d\,e + \&c.$ then $L = \mathcal{D}$, $N = A$, $m = 4$, $Z = \overline{a\,b\,c\,d - a\,b\,c\,e}|^2 + \overline{a\,b\,c\,d - b\,c\,d\,e}|^2 + \overline{a\,b\,c\,d - a\,c\,d\,e}|^2 + \&c. = \alpha$,

$\beta =$

$\beta = 0 = \gamma = \delta$, therefore $\dfrac{\overline{4 \times 5 - 4}}{\overline{4 + 1 \times 5 - 4 + 1}} \times E^2$ or

$\dfrac{2}{5} E^2$ exceeds $\mathcal{D} \times A$ by $\dfrac{\overline{5 + 1}}{\overline{4 + 1 \times 5 - 4 + 1}} \times Z -$

$\dfrac{1}{2} \alpha = \dfrac{3}{5} Z - \dfrac{1}{2} \alpha = \dfrac{1}{10} Z = \dfrac{1}{10} \times \overline{abcd - abce}|^2 +$

$\dfrac{1}{10} \times \overline{abcd - bcde}|^2 + \&c.$ which is a pofitive

Number when the Roots are real Numbers.

PROPOSITION II.

Let $x^n - B x^{n-1} + C x^{n-2} - \mathcal{D} x^{n-3} + E x^{n-4} -$
$\&c. \pm A = 0$ be an Æquation of any Degree, whofe
Roots with their Signs let be expreffed by the Let-
ters a, b, c, d, e, f, $\&c.$ let M reprefent any Coeffi-
cient of this Æquation, L, N the Coefficients adja-
cent to M; K, O the Coefficients adjacent to L, N;
I, P thofe adjacent to K, O; H, $\mathcal{Q}$ thofe adjacent
to I, P, and fo on. Let m reprefent the Exponent
of M and let Z (as in the preceeding Propofition)
reprefent the Sum of the Squares of all the poffi-
ble Differences between the Terms of the Coeffici-
ent M. Then the Product of the Square of any

Coefficient M multiply'd by the Fraction $\dfrac{1}{2} \times$

$$1 - \dfrac{1}{n \times \dfrac{\overline{n-1}}{2} \times \dfrac{\overline{n-2}}{3} \times \&c. \times \dfrac{\overline{n-m+1}}{m}} \text{ doth}$$

always

always exceed $L \times N - K \times O + I \times P - H \times Q + \&c.$

by $\dfrac{\frac{1}{2} Z}{n \times \dfrac{n-1}{2} \times \dfrac{n-2}{3} \times \&c. \times \dfrac{n-m+1}{m}}$ which

is always a pofitive Number, when the Roots a, b, c, d, e $\&c.$ are real Numbers pofitive or negative. Let the Æquation be of the feventh Degree or $x^7 - B x^6 + C x^5 - D x^4 + E x^3 - F x^2 + G x - A = 0$, whofe Roots let be a, b, c, d, e, f, g, in which Cafe $n = 7$. Let $M = E = abcd + abce + abcf + abcg + bcde + \&c.$ then $m = 4$, $L = -D$, $N = -F$, $K = C$, $O = G$, $I = -B$, $P = -A$, $Z = \overline{abcd - abce}|^2 + \overline{abcd - abcf}|^2 + \overline{abcd - abcg}|^2 + \&c.$ Therefore $\dfrac{1}{2} \times$

$\overline{1 - \dfrac{1}{7 \times \dfrac{6}{2} \times \dfrac{5}{3} \times \dfrac{4}{4}}} \times E^2$ or $\dfrac{17}{35} E^2$ exceeds $D \times$

$F - C \times G + B \times A$ by $\dfrac{\frac{1}{2} Z}{7 \times \dfrac{6}{2} \times \dfrac{5}{3} \times \dfrac{4}{4}}$ or $\dfrac{Z}{70} =$

$\dfrac{1}{70} \times \overline{abcd - abce}|^2 + \dfrac{1}{70} \times \overline{abcd - abcf}|^2 + \&c.$

From this Propofition, is deduced the following Rule for determining the Number of impoffible Roots in any given Æquation. From each of the Unciæ of the middle Terms of that Power of a Binomial, whofe

whofe Index is the Dimenfions of the propofed Æqua-
tion, fubtract Unity, then divide each Remainder by
twice the Correfpondent *Uncia*, and fet the Fracti-
ons which refult from this Divifion, above the mid-
dle Terms of the given Æquation. And under any
of the middle Terms if its Square multiplyed by the
Fraction ftanding above it, be greater than the Rect-
angle under the immediately adjacent Terms, *Minus*
the Rectangle under the next adjacent Terms, *Plus*
the Rectangle under the Terms then next adjacent
— &c. place the Sign $+$, but if it be lefs, place
the Sign —. And under the firft and laft Term
place $+$. And there will be at leaft as many im-
poffible Roots, as there are Changes in the Series of
the under-written Signs from $+$ to —, or from —
to $+$. Let it be required to determine the Num-
ber of impoffible Roots in the Æquation x^7 —
$5x^6 + 15x^5 - 23x^4 + 18x^3 + 10x^2 -$
$28x + 24 = 0$. The *Unciæ* of the middle Terms
of the 7th Power of a Binomial are 7, 21, 35, 35,
21, 7, from which fubtracting Unity, and dividing
each of the Remainders by twice the correfpondent

Uncia, the Quotients will be $\dfrac{6}{14}$, $\dfrac{20}{42}$, $\dfrac{34}{70}$,

$\dfrac{34}{70}$, $\dfrac{20}{42}$, $\dfrac{6}{14}$ or $\dfrac{3}{7}$, $\dfrac{10}{21}$, $\dfrac{17}{35}$, $\dfrac{17}{35}$,

$\dfrac{10}{21}$, $\dfrac{3}{7}$ which Fractions place above the middle

Terms of the Æquation, has $x^7 \overset{\frac{3}{7}}{-} 5x^6 + \overset{\frac{10}{21}}{15}x^5 -$
$$\qquad + \qquad - \qquad +$$
$$\text{B b b b} \qquad 23x^4 +$$

$\overset{\frac{12}{35}}{23x^4} + \overset{\frac{12}{35}}{18x^3} + \overset{\frac{10}{21}}{10x^2} - \overset{\frac{4}{7}}{28x} + 24 = 0.$ Then

$\quad - \qquad + \qquad - \qquad + \qquad +$

becaufe the Square of $-5x^6$ multiply'd into the

Fraction over its Head $\dfrac{3}{7}$, to wit $\dfrac{75}{7} x^{12}$ is lefs

than $x^7 \times 15 x^5$ or $15 x^{12}$ I place the Sign $-$ un-
der the Term $5x^6$. Becaufe the Square of $15x^5$

multiply'd by the Fraction over its Head $\dfrac{10}{21}$

to wit $\dfrac{705}{7} x^{10}$ is greater than $\overline{-5x^6 \times -23x^4} -$

$\overline{x^7 \times 18 x^3} = 97 x^{10}$ I place the Sign $+$ under

the Term $15 x^5$. Seeing $\dfrac{8993}{35} x^8$ (the Square of the

Term $-23 x^4$ multiply'd by the Fraction over

its Head $\dfrac{17}{35} \Big)$ is lefs than $15 x^5 \times 18 x^3 -$

$\overline{-5x^6 \times 10 x^2} + \overline{x^7 \times -28 x} = 292 x^8$, I place
the Sign $-$ under the Term $23 x^4$. Becaufe

$\overline{18 x^3}\big|^2 \times \dfrac{17}{35}$ or $\dfrac{5508}{35} x^6$ exceeds $\overline{-23 x^4 \times 10 x^2} -$

$\overline{15 x^5 \times -28 x} + \overline{-5 x^6 \times 24} = 70 x^6$ I place
the Sign $+$ under the Term $18 x^3$. Since $\overline{10 x^2}\big|^2 \times$

$\dfrac{10}{21}$ or $\dfrac{1000}{21} x^4$ is lefs than $\overline{+18 x^3 \times -28 x} -$

$\overline{-23 x^4 \times 24} = 48 x^4$ I place the Sign $-$ under
the

the Term $10x^2$. Becaufe $\overline{28x}|^2 \times \dfrac{3}{7}$ or $336x^2$ is greater than $10x^2 \times 24 = 240x^2$ under $28x$ I place $+$, then under the firft and laft Terms I place $+$; and the fix Changes of under-written Signs fhews that there are fix impoffible Roots.

If the impoffible Roots were to be found by the *Newtonian* Rule, the Operation would ftand thus:

$$x^7 \overset{\tfrac{2}{7}}{-} 5x^6 \overset{\tfrac{4}{9}}{+} 15x^5 \overset{\tfrac{3}{4}}{-} 23x^4 \overset{\tfrac{2}{3}}{+} 18x^3 \overset{\tfrac{5}{9}}{+} 10x^2 -$$
$$+ \quad - \quad + \quad + \quad + \quad +$$

$$\overset{\tfrac{3}{7}}{28}x + 24 = 0,$$ by which Rule there are found
$$+ \qquad +$$

only two impoffible Roots, whereas there are fix to wit $1 + \sqrt{-3}$, $1 - \sqrt{-3}$, $1 + \sqrt{-2}$, $1 - \sqrt{-2}$, $1 + \sqrt{-1}$, $1 + \sqrt{-1}$, the feventh Root being -1.